COMSATS Institute of Information Technology, Islamabad
Department of Mathematics
Assignment # 1

Program: MS/PhD	Instructor: Dr. M Saeed Akram
Semester:	Deadline: Oct 21, 2016

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1. Call a subset O in plane $\mathbb{R}^2$ *open* provided for each $(x, y) \in O$, there is a $r > 0$ for which the Rectangle $(x - r, x + r) \times (y - r, y + r)$ is contained in O.
   (a) Does an analouge of Proposition 1.4.9 hold for open sets in plane $\mathbb{R}^2$? Justify your answer.
   (b) Does the Heine Borel Theorem hold for $\mathbb{R}^2$? Justify your answer.
2. Let $\mathbb{R}^e$ denotes the the set of extended reals numbers i.e., $\mathbb{R}^e = \mathbb{R} \cup \{-\infty, \infty\}$. Call a subset O in $\mathbb{R}^e$ *open* if for each $x \in O$ either there exist $r > 0$ such that $(x - r, x + r) \subset O$ or there exists $b \in \mathbb{R}$ such that $x \in [-\infty, b)$ is contained in O or there exists $a \in \mathbb{R}$ such that $x \in (a, \infty]$ is contained in O.
   Does an analouge of Proposition 1.4.9 hold for open sets in $\mathbb{R}^e$? Justify your answer.
3. Prove that
   (a) $\cup_{n=1}^{\infty}[a + 1/n, b - 1/n] = (a, b)$ for any $a, b \in \mathbb{R}$.
   (b) $\cap_{n=1}^{\infty}(a - 1/n, b + 1/n) = [a, b]$ for any $a, b \in \mathbb{R}$.
   (c) Would (a) and (b) still hold if $a, b \in \{\infty, -\infty\}$? Justify your answer.
4. If a set has measure zero, is it also countable? Justify your anawer.
5. If a set is measurable, is it also Borel? Justify your answer.