



COMSATS Institute of Information Technology, Islamabad  
Department of Mathematics  
Assignment # 3

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Program: MS/PhD  
Semester:

Instructor: Dr. M Saeed Akram  
Deadline: Dec 5, 2016

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1. Let  $p \geq 1$  be a fixed real number. By definition, each element in the space  $l^p(\mathbb{Z})$  is a sequence  $x = (x_i)_{i \in \mathbb{Z}} = (\dots, x_{-1}, x_0, x_1, \dots)$  of numbers such that  $\dots + |x_{-1}|^p + |x_0|^p + |x_1|^p + \dots$  converges; thus

$$\sum_{i=-\infty}^{\infty} |x_i|^p < \infty \quad (p \geq 1, \text{ fixed})$$

and  $d$  is defined by

$$d(x, y) = \left( \sum_{i=-\infty}^{\infty} |x_i - y_i|^p \right)^{1/p}.$$

Is  $(l^p(\mathbb{Z}), d)$  a metric space? Justify your answer.